INVESTMENT TALENT AND THE PARETO WEALTH DISTRIBUTION: THEORETICAL AND EXPERIMENTAL ANALYSIS

Moshe Levy and Haim Levy*

Abstract—The empirically documented Pareto wealth distribution at high wealth levels implies rather extreme wealth inequality. Is this inequality primarily due to differential talent, or is it due to luck? The answer to this question has profound political, social, and philosophical implications, as well as implications regarding market efficiency. We address this question theoretically and with a unique investment experiment with equal initial endowments and real out-of-pocket money. We show that the empirically observed Pareto distribution implies that luck, rather than differential investment talent, is the main force driving inequality at high wealth levels.

I. Introduction

WEALTH distribution and the source of inequality are important issues with many implications. At the end of the nineteenth century, the Italian-born Swiss economist Vilfredo Pareto (1897) discovered that in the high-wealth range the population's wealth (and income) are distributed according to a particular functional form—a power function. The parameters of this distribution may change across societies, but regardless of the social or political conditions, taxation, and the like, Pareto claimed that the wealth distribution obeys this general distribution law, which is named after him, Pareto's law or Pareto's distribution. Regarding the fit of income distributions to the Pareto distribution, Blinder (1974) asserts:

> It may well be no accident that the upper tails of almost all income distributions, where returns to capital dominate and earnings play a minor role, exhibit a striking resemblance to the Pareto distribution (pp. 7–8).

The discovery of a universal mathematical law for the distribution of wealth has led to different theories about the origins of this wealth distribution, and in particular, the wealth inequality which it implies. Obviously, these theories have tremendous social and philosophical implications. The first to suggest an explanation for the Pareto distribution of wealth was Pareto himself (Pareto, 1906). Pareto suggested that the distribution of wealth corresponds to an underlying distribution of human abilities. However, Pareto did not offer a mathematical model that would explain the distribution of abilities and its relation to his law. Pareto's explanation was extended by Davis (1941), who introduced the "law of the distribution of special abilities" which asserts that the probability of obtaining an additional unit of ability is independent of the level of ability. This model, however, leads to a normal distribution of ability and therefore presumably to a normal, rather than a Pareto, distribution of wealth. A different model for the distribution of ability was formulated by Boissevain (1939), who considered that the distribution of abilities could be represented as a product of several factors, each of which follows a binomial distribution. Boissevain's model explains the positive skewness in the distributions of wealth and income, but leads to a log normal wealth distribution, not to the empirically observed Pareto distribution.

The main models that offer an explanation for the precise form of the Pareto wealth distribution are the Markov chain model of Champernowne (1953), the stream model of Simon (1955), and the birth-and-death model of Wold and Whittle[1] (1957). Although these models are quite different from each other in their details, they have three common factors: they are all based on a stochastic multiplicative process of wealth accumulation, they impose some *lower bound* on wealth, and they all assume *homogeneous* wealth accumulation talent. Thus, in all the present models which can explain the empirical Pareto wealth distribution, the only reason for inequality is the stochastic process—chance. This implies that there is no differential ability in asset selection or in timing the market, which is in line with the efficient-market hypothesis.

The above models for the process of wealth accumulation, and their provocative implication that chance, rather than talent, is the main reason for inequality at high wealth levels, encounter two major objections:

(a) To obtain the precise Pareto distribution, one needs an incredibly long time (Shorrocks, 1973; Blinder, 1974). It is natural to ask: how long (how many generations) will it take a new economy with a uniform distribution of wealth before Pareto's law emerges? For example, suppose that there had been perfect equality in the Soviet Union before it adopted a capitalist economy (which is obviously not really the case). How long would we need to wait before the initial equal wealth distribution transformed to the Pareto distribution with the extreme inequality which it implies? Of course, it may be that one needs a very long period to obtain the *precise* Pareto distribution, but only a short period to obtain an *approximate* Pareto distribution.

(b) Many find it unreasonable to assume homogeneous investment talent. Indeed, most money manager compensation schemes implicitly assume that differential talent exists; hence the compensation is directly

Received for publication July 12, 2001. Revision accepted for publication May 20, 2002.

* The Hebrew University of Jerusalem.

The authors thank the two anonymous referees for their helpful comments and suggestions. They also thank the Krueger Center of Finance and the Zagagi Fund for financial support.

[1] For a historical review of the Pareto distribution see Persky (1992). Reviews of models generating the Pareto distribution can be found in Stiendl (1965), Blinder (1974), Arnold (1983), and Slottje (1989).

The Review of Economics and Statistics, August 2003, 85(3): 709–725

linked to performance. Obviously, there is no point in performance-based compensation if the performance is primarily due to luck.

In this paper we attempt to bridge between two areas of research: the distribution of wealth, which is mainly in the economic research arena, and market efficiency and investment talent, which are fundamental research topics in finance. We first show theoretically that the Pareto wealth distribution is consistent only with a very limited degree of differential investment talent. Thus, the empirically documented Pareto wealth distribution (in the high wealth range) limits the degree of differential investment talent in the market, in line with the notion of market efficiency. We show that this, in turn, implies that the main reason for wealth inequality (in the high wealth range) is chance, rather than differential investment talent.

Our theoretical results show that a precise Pareto wealth distribution is obtained only when the number of investment periods is infinite, and in the absence of differential investment talent. To complement the theoretical results and to test whether an approximate Pareto wealth distribution can emerge within a reasonable time from a realistic capital investment process, in which some differential investment talent may exist, we have conducted an experimental study. At the beginning of the experiment each subject is allocated the same amount of money. The subjects trade in a stock market, and the wealth they accumulate is a function of their investment success [that is, we focus on a situation in which, as Blinder (1974, p. 7) puts it, "returns to capital dominate and earnings play a minor role"]. We record the wealth of each subject in each trading period, and we analyze the wealth distribution in each period. In addition, we directly estimate the degree of differential investment talent across investors.

There are five main goals for the experimental part of our study:

(1) To examine whether a Pareto or a log normal wealth distribution is obtained when wealth is accumulated solely in the capital market, where initially each subject has an equal amount of wealth.[2]
(2) To examine the speed of convergence to Pareto's law, if it indeed occurs.
(3) To examine the magnitudes of the Pareto distribution parameters. We also examine the changes in these parameters across time, emphasizing their direction and speed.
(4) To test whether there is substantial differential investment talent across investors.
(5) And finally, to test directly whether the inequality of terminal wealth is primarily due to differences in investment talent across investors or due to chance. Did the wealthier subjects in the experiment invest more wisely, or were they simply lucky?

Of course, analyzing such issues and, in particular, having an initial uniform distribution of wealth is possible only in laboratory experiments, which is the framework of this paper. A unique feature of this experiment is that the subjects participating could gain *or lose* out-of-pocket money, which makes the investment environment in our experiment very realistic.

The structure of this paper is as follows. In section II we theoretically analyze the relationship between the two main issues investigated: the Pareto wealth distribution and differential investment talent. In section III, we describe the experiment. In section IV we provide the main experimental findings. Section V concludes the paper with a discussion of the main results and their implications.

II. The Pareto Distribution and Differential Talent

In section IIA below we show that general wealth accumulation processes with homogeneous investment talent lead to a Pareto wealth distribution. In section IIB we demonstrate that a substantial talent differential leads to a wealth distribution which is significantly different from the Pareto distribution. That subsection also provides a numerical estimation of the degree of differential talent which is still statistically consistent with the empirically observed Pareto distribution.

A. *Theoretical Analysis*

When examining the wealth distribution in society one typically finds two distinct regions. In the lower wealth range the distribution of wealth can be approximated by the log normal distribution. In the high range the distribution is described by the Pareto distribution (see, for example, Stiendl, 1965; Atkinson & Harrison, 1978; and Persky, 1992). In this paper we focus on the Pareto distribution which characterizes the high wealth range. This range is extremely important because, although it accounts for a small part of the population (typically about 5%), it accounts for most of the wealth.[3] In addition, when considering wealth accumulation through capital investments, it is the high wealth range which is relevant to the analysis.

The main models suggested in order to explain the empirically observed Pareto distribution are the models by Champernowne (1953), Simon (1955), and Wold and Whittle (1957). Whereas each of the above three models makes

[2] We use the term "wealth" to mean the experimental wealth. Obviously, subjects do not have the same nonexperimental wealth. Several researchers have shown that in experimental situations subjects typically conduct "mental accounting" and consider primarily their experimental wealth in their decision-making, ignoring other sources of wealth (see, for example, Thaler, 1985, and Levy, 1994).

[3] According to Wolff (1996), the top 1% of the population in the United States holds more than 40% of the total wealth. Díaz-Giménez, Quadrini, and Ríos-Rull (1997) report that the top 5% of the population hold about 55% of the total wealth.

some specific assumptions regarding the process of wealth accumulation, we suggest here a generalization of them which demonstrates that the key element required to ensure the Pareto distribution is homogeneous talent. The process of wealth accumulation can be generally formulated as

$$X_i^{t+1} = X_i^t \tilde{\lambda}_i^t + \tilde{S}_i^t - \tilde{C}_i^t, \tag{1}$$

where X_i^t denotes the wealth of investor i at the beginning of period t, $\tilde{\lambda}_i^t$ is a stochastic return on investor i's investment at time t, and $\tilde{S}_i^t$ and $\tilde{C}_i^t$ are the period t salaries and consumption, respectively. For low-wealth individuals the salary and consumption components are typically the dominant elements in equation (1), whereas for high-wealth individuals the capital investment component is typically dominant (see Wolff, 2000). Indeed, the models of Champernowne, Simon, and Wold and Whittle, which model the Pareto distribution characterizing the high wealth range, abstract from labor income and consumption and focus only on the capital investment process.[4] In this approach one states that the model only deals with the wealth distribution among individuals with wealth exceeding some lower bound, and neglects the income and consumption components to obtain

$$X_i^{t+1} = X_i^t \tilde{\lambda}_i^t, \tag{2}$$

This is the approach we adopt here. This approach can be interpreted as stating that when the wealth of an investor reaches some lower bound, he/she withdraws (or goes bankrupt) and does not continue to invest in the capital market. The assumption of a sharp boundary between investors and noninvestors is a proxy for the more gradual increase in the significance of capital investment as a function of wealth. This assumption simplifies the model, but it can be relaxed. In fact, results similar to those presented below can be obtained with nonproportionate growth in wealth as described by equation (1) and no lower bound (see Kesten, 1973).

The crucial element ensuring the convergence of the wealth distribution to the Pareto distribution is the assumption of homogeneous investment talent. In our context, homogeneous investment talent means that all investors draw their returns randomly from the same distribution (the *realized* return, however, generally differs from one investor to another). That is, the return $\tilde{\lambda}_i^t$ is drawn from the same distribution $g(\lambda)$ for all investors.[5] Notice though that $\tilde{\lambda}_i^t$ is a return realization which is specific to investor i. Thus, homogeneous talent implies that it is only the randomness of the process which discriminates between investors [in contrast, differential investment talent would imply $g_i(\lambda) \neq g_j(\lambda)$ for $i \neq j$].

The Pareto distribution is obtained as a limit distribution of the process (2) under two alternative assumptions:

(a) The process has a negative drift, that is, $E[\tilde{\lambda}] < 1$, and the lower bound on wealth is stated in terms of absolute wealth.
(b) The process has a positive drift, that is, $E[\tilde{\lambda}] > 1$, and the lower bound is stated in terms of the average wealth.

Most previous models take approach (a) and assume a negative drift. However, as returns on capital investments are on average positive, in theorem 1 below we take approach (b). Thus, theorem 1 generalizes the previous results to the case in which the average return is positive.[6] The theorem also shows the link between the absence of differential talent and the Pareto wealth distribution.

Theorem 1. For any nondegenerate initial wealth distribution and nontrivial return distribution, the wealth accumulation process given by equation (2) with a lower bound leads to convergence of the wealth distribution to the Pareto distribution.

Proof. Define the *normalized* wealth of investor i as $x_i^t = X_i^t / \sum_j X_j^t$, that is, the investor's wealth as a fraction of the total wealth (note that normalized wealth is denoted by lowercase x, whereas uppercase X denotes absolute wealth in dollars). We will prove that the distribution of the normalized wealth converges to the Pareto distribution. As will be explained below, this implies that the actual wealth distribution is also a Pareto distribution, and with the same exponent, because at any given time the actual wealth is simply the normalized wealth multiplied by a constant.

The process (2) implies that the normalized wealth of each investor changes from x_i^t at time t to $x_i^t \tilde{\lambda}_i^{*t}$ at time $t +$

[4] Neoclassical growth models with earning shocks and endogenously determined consumption generally succeed in explaining the lower and central part of the wealth distribution quite well, but cannot explain the high wealth concentration at the Pareto right tail (see, for example, Auerbach & Kotlikoff, 1987; Aiyagari, 1994; Ríos-Rull, 1995; Huggett, 1996; and Castaneda, Díaz-Giménez, & Ríos-Rull, 1997). We also abstract from the effects of social security [see Feldstein and Pellechio (1979) for these effects].

[5] All investors draw an observation at random from the same distribution $g(\lambda)$. However, this does not mean that all investors hold the same portfolio. More realistically, investors face distributions $h_1(\lambda)$, $h_2(\lambda)$, . . . , $h_M(\lambda)$ (that is, M possible distributions, which consist of individual stocks or portfolios), and each investor makes a subjective assessment by which he/she decides in which distribution to invest. This assessment may change in each period due to macroeconomic or firm-specific factors as estimated by the investor. If investors have no talent, they switch randomly among these M distributions. When the number of investment periods is large, this is equivalent to asserting that all investors select from the same *metadistribution,* which we denote by $g(\lambda)$.

[6] For a similar analysis see Levy (forthcoming). Recently, Gabaix (1999) employed a similar approach to explain Zipf's (1949) law for the distribution of city sizes. Note that Zipf's law is a special case of Pareto's law with an exponent $\alpha = 0$. To obtain this exponent Gabaix assumes a lower bound (on city size) which approaches 0, which leads to the result of $\alpha = 0$. In contrast, in the context of the wealth distribution we would expect a finite lower bound, because investors with low wealth do not tend to be heavily invested in the capital market. With a finite positive lower bound we obtain $\alpha > 0$, which conforms with the empirically observed α (typically in the range 1–2).

1, that is, $x_i^{t+1} = x_i^t \tilde{\lambda}_i^{*t}$, where λ_i^{*t} is defined as $\lambda_i^{*t} \equiv \lambda_i^t(\Sigma_j X_j^t/\Sigma_j X_j^{t+1})$. Denote the cumulative normalized wealth distribution at time t and at time $t+1$ by $F(x, t)$ and $F(x, t+1)$, respectively. The latter is given by

$$F(x, t+1) = \int_0^{+\infty} F\left(\frac{x}{\lambda^*}, t\right) g(\lambda^*)\, d\lambda^*, \tag{3}$$

where all values of λ^* such that the wealth at $t+1$ is equal to $(x/\lambda^*)\lambda^* = x$ are accounted for.[7]

Equation (3) describes a process in which the probability $F(x)$ at time $t+1$ is a weighted average of the probability at points surrounding x (points x/λ^*) at time t. Thus, starting from an arbitrary probability density, $F(x, 0)$, the distribution $F(x, t)$ undergoes a continuous smoothing process. In the presence of an effective lower bound on wealth ($x_i^t \geq x_{\min}$), this smoothing process is analogous to diffusion towards a barrier (Mandelbrot, 1960; Levy & Solomon, 1996). Such a process is well known to lead to the convergence of $F(x, t)$ to a stationary distribution (see, for example, Boltzmann, 1964; Feynman, Leighton, & Sands, 1964). For the limiting stationary wealth distribution we have $F(x, t+1) = F(x, t) = F(x)$, and equation (3) becomes

$$F(x) = \int_0^{+\infty} F\left(\frac{x}{\lambda^*}\right) g(\lambda^*)\, d\lambda^*. \tag{4}$$

Differentiating with respect to x, we obtain the density function:

$$f(x) = \int_0^{+\infty} f\left(\frac{x}{\lambda^*}\right) \frac{1}{\lambda^*}\, g(\lambda^*)\, d\lambda^*. \tag{5}$$

Let us proceed to show that the Pareto distribution is the solution to equation (5). The Pareto density function is given by

$$f(x) = \frac{\alpha k^\alpha}{x^{\alpha+1}} \qquad (\alpha > 0, \quad x \geq k > 0), \tag{6}$$

where α and k are constants, and k is the lower limit of x ($k = x_{\min}$; see, for example, Johnson & Kotz, 1970, chapter 19). In order to show that the Pareto distribution is a solution to equation (5), substitute the Pareto probability density function [equation (6)] for $f(x)$ in equation (5) to obtain

$$\frac{\alpha k^\alpha}{x^{\alpha+1}} = \int_0^{+\infty} \frac{\alpha k^\alpha}{(x/\lambda^*)^{\alpha+1}} \frac{1}{\lambda^*}\, g(\lambda^*)\, d\lambda^*$$

$$= \frac{\alpha k^\alpha}{x^{\alpha+1}} \int_0^{+\infty} \lambda^{*\alpha} g(\lambda^*)\, d\lambda^*. \tag{7}$$

Thus, it is evident that the Pareto distribution with α satisfying $\int_0^{+\infty} \lambda^{*\alpha} g(\lambda^*)\, d\lambda^* = 1$ is a solution to equation (5). It can also be shown that the Pareto distribution is the unique solution.[8]

Hence, the normalized wealth distribution converges to the Pareto distribution. As the actual wealth at any given time is simply the normalized wealth times a constant, the actual wealth distribution is also a Pareto distribution with the same exponent. To see this note that $f(x) = \alpha k^\alpha / x^{\alpha+1}$ and that $X = Cx$, where C is a constant. Thus, the actual wealth distribution is given by

$$h(X) = f(x)\frac{\partial x}{\partial X} = f\left(\frac{X}{C}\right)\frac{1}{C} = \frac{\alpha k^\alpha}{X^{\alpha+1}} \frac{C^{\alpha+1}}{C} = \frac{\alpha (kC)^\alpha}{X^{\alpha+1}},$$

which is a Pareto distribution with the same exponent α, and with a lower wealth bound kC. Q.E.D.

Note that, because we are dealing with a process with a positive drift, the entire wealth distribution drifts to the right as C grows over time. Thus, although the distribution of *normalized* wealth converges to a stable Pareto distribution, the actual wealth distribution converges to a Pareto distribution with the same α, but with minimum wealth, average wealth, and variance that grow over time.

Theorem 1 shows that the Pareto distribution is a limit distribution of the wealth accumulation process (2), in which wealth differentiation is driven only by luck. We would like to emphasize that the analysis is quite general and does not rely on any specific form of the return distribution $g(\lambda)$, as long as this distribution is nontrivial. Note that, without the lower limit $x_{\min}$ on wealth, by the central limit theorem, equation (2) would lead the distribution $f(x, t)$ to converge to the log normal distribution. However, with the lower bound $x_{\min}$, we do not simply get the right-hand tail of the log normal distribution, but rather a different distribution—Pareto, with relatively high density near the lower bound due to the boundary condition.[9]

Theorem 1 above states that a sufficient condition for the convergence of the wealth distribution to the Pareto distribution is that the wealth differentiation is driven only by luck. It can be shown that this condition is not only suffi-

[7] As $\tilde{\lambda}$ represents the total return on capital investment, it cannot be less than 0 (which corresponds to a rate of return of −100%), and thus the same is true for $\tilde{\lambda}^*$.

[8] The uniqueness of the Pareto distribution as a limiting distribution of this process and as a solution to equation (5) results from the fact that the only positive g-harmonic functions on $\Re$ are exponentials (see Choquet, 1960; Loomis, 1962; and Furstenberg, 1965, theorem B, p. 291).

[9] Note that with $x_{\min} \to 0$ the Pareto exponent α converges to 0. Now $\alpha \to 0$ implies a Pareto distribution which converges to the tail of a log normal distribution. Thus, the lower the bound, the closer we get to the log normal.

cient, but also necessary, to ensure the Pareto distribution (see Levy (forthcoming) and the discussion in the next section). In other words, a process with substantial differential investment talent does not lead to the Pareto distribution. Hence, there is a strong link between homogeneous investment talent (which would be expected in an efficient market), and the Pareto wealth distribution. As there is an excellent fit between the empirical wealth distribution and the Pareto distribution, this empirical observation suggests the lack of substantial differential investment talent.

Several researchers empirically do find evidence of some differential investment talent (see, for example, Sharpe, 1966, and Chevalier and Ellison, 1999). Can these findings be consistent with the Pareto distribution? In other words, what degree of differential investment talent can still be statistically consistent with the empirically observed Pareto distribution? To this issue we turn next.

TABLE 1.—COMPARISON OF THE WEALTH DISTRIBUTION IN A HETEROGENEOUS TALENT POPULATION WITH THE PARETO AND LOG NORMAL DISTRIBUTIONS[a]

Mean Return of		Observed Statistic[b]	
Population 1 (μ_1)	Population 2 (μ_2)	D_P	D_L
1.12	1.08	0.182	0.204
1.12	1.09	0.188	0.210
1.12	1.10	0.144	0.195
1.12	1.11	0.025	0.152
1.12	1.12	0.017	0.155
1.12	1.13	0.097	0.182
1.12	1.14	0.193	0.215
1.12	1.15	0.184	0.210
1.12	1.16	0.170	0.202
1.12	1.17	0.147	0.193
1.12	1.18	0.132	0.182
1.12	1.19	0.140	0.190
1.12	1.20	0.193	0.210

[a] For population 1, returns are drawn randomly from a normal distribution with $\mu_1 = 12\%$ and $\sigma = 20\%$. For population 2, returns are drawn randomly from a normal distribution with mean μ_2 and $\sigma = 20\%$. We report the Kolmogorov-Smirnov statistic for different values of μ_2.

[b] D_P and D_L are the Kolmogorov-Smirnov statistics corresponding to the Pareto and log normal distributions respectively. With 1600 observations, the critical Kolmogorov-Smirnov value at the 1% confidence level is $D_{1\%} = 0.041$, at the 5% level it is $D_{5\%} = 0.034$, and at the 20% level it is $D_{20\%} = 0.027$. The highlighted cases are those in which the Pareto distribution cannot be rejected.

B. The Maximal Talent Differential Which is Consistent with a Pareto Distribution

Although the Pareto wealth distribution cannot precisely hold with a substantial investment talent differential, it can be consistent with some degree of differential talent, in the sense that the Pareto distribution cannot be statistically rejected and it fits the data better than any other candidate distribution. In this section we allow for differential talent, and we employ numerical simulations to examine the degree of differential talent which may still be consistent with the Pareto distribution. Namely, we increase the talent differential until the hypothesis that wealth is distributed according to the Pareto distribution is rejected by the Kolmogrov-Smirnov goodness-of-fit statistic. Thus, we determine the maximum possible talent differential, given that the Pareto distribution does empirically prevail.

Let us elaborate. Suppose that there are two populations with different investment talents. The investors of one group invest in a risky asset with a mean rate of return of, say, 12% and a standard deviation of 20%, and the investors of the second group invest in a risky asset with mean rate of return μ and a standard deviation of 20%. Both return distributions are normal, and the investors obtain random returns drawn from these two distributions, respectively. Within each group talent is homogeneous, as all investors in the group draw their returns from the same distribution. If $\mu < 12\%$, the first group is more talented than the second, if $\mu = 12\%$, there is no differential talent, and if $\mu > 12\%$, the second group is more talented than the first. We fix the standard deviation at 20%, and hence analyze the stock selection ability adjusted for risk. We run a simulation with 800 investors in group 1 and 800 in group 2. Each investor draws an observation from the corresponding distribution, and his/her wealth is measured after 200 investment periods by $\tilde{X}_i^{200} = \prod_{t=1}^{200} (1 + \tilde{R}_i^t)$, where $\tilde{R}_i^t$ is the rate of return drawn by investor i at period t. The lower wealth bound is set at 10% of the average wealth.[10] Theorem 1 reveals that with no differential investment talent and an infinitely large number of investment periods, the wealth distribution is either Pareto (with a lower wealth bound) or log normal (with no lower wealth bound). Thus, the Pareto and the log normal distributions are natural candidates to fit the wealth distribution in the simulations (and later on in the experiment). The wealth distribution of all 1600 investors after 200 investment periods was measured for goodness of fit to the Pareto distribution as well as to the log normal distribution. To test the hypothesis that the wealth distribution is not significantly different from the Pareto distribution or the log normal distribution we use the Kolmogrov-Smirnov statistic given by

$$D = \max\,[F_{\text{Ex}}(X) - F_{\text{Th}}(X)],$$

where $F_{\text{Ex}}(X)$ and $F_{\text{Th}}(X)$ are the experimental and the theoretical (Pareto or log normal) cumulative distributions, respectively. Table 1 reports the results. With 1600 observations the critical Kolmogorov-Smirnov value at the 1% confidence level is $D_{1\%} = 1.63/\sqrt{1600} = 0.041$, and at the 5% level it is $D_{5\%} = 1.36/\sqrt{1600} = 0.034$. Thus, if the observed sample value D is greater than these critical values, the null hypothesis asserting that there is a good fit of the simulated wealth distribution to the theoretical distribution is rejected. Table 1 reveals that the hypothesis that wealth is log-normally distributed is strongly rejected, regardless of the talent differential. Compare D_L (where L stands for log normal) of about 0.15–0.21, given in table 1, with the critical value $D_{1\%} = 0.041$. Thus, the log normal distribution is rejected even with homogeneous talent.

[10] In order to keep the number of investors in the simulation constant, whenever an investor crosses the lower wealth bound and exits the market we introduce a new investor with the minimal wealth. Similar results are also obtained when the new investor's wealth is determined stochastically.

A completely different result is obtained for the Pareto distribution (see D_P in table 1, where P stands for Pareto). With no differential talent, that is, $\mu = 12\%$ for both groups, we obtain $D_P = 0.017 < D_{5\%} = 0.034$; hence the null hypothesis asserting that wealth is distributed according to Pareto's law cannot be rejected at the 5% confidence level (and, of course, cannot be rejected at a lower level, such as 1%). Moreover, the fit is so strong that we cannot reject this hypothesis even at the 20% level, as the critical value is $D_{20\%} = 0.027$, which is larger than the observed value $D_P = 0.017$. When we allow a 1% talent differential, that is, a 1% difference in mean returns with $\mu_1 = 12\%$ and $\mu_2 = 11\%$, we obtain $D_1 = 0.025$; hence once again the Pareto distribution hypothesis is not rejected even at the 20% level, let along at lower confidence levels. However, when we increase the differential talent to more than 1%, the Pareto distribution is rejected even at the 1% level, let alone at higher levels.

This result is quite general, and we obtain it for various rate-of-return distributions and for different specifications of market compositions (more than two investor types, and so on). For instance, we also examine the case where talent is distributed randomly in the population.[11] As before, all investors draw returns from distributions with standard deviation 20%, but the mean return μ is different across investors. For each investor μ is drawn from a normal distribution with average 12% and standard deviation σ_μ. Table 2 reports the fit of the wealth distribution generated after 200 periods to the Pareto and log normal distributions as a function of σ_μ, which is a measure of the talent differential. Note that $\sigma_\mu = 0$ implies homogeneous talent. Table 2 reveals that the log normal distribution is always rejected. However, the Pareto distribution cannot be rejected (even at the 20% level) when σ_μ is 0.25% or smaller. These results conform with those of table 1, because $\sigma_\mu = 0.25\%$ implies that 95% of the investors have μ within a range of about 1%.

Thus, accepting the fact that empirical data reveal a good fit, albeit not a perfect one, to the Pareto distribution, we find that not rejecting the Pareto distribution is possible even with a 1% differential in mean returns (this talent differential is consistent with the empirical findings of Chevalier and Ellison, 1999). This cap on differential investment talent allows us to get an idea of the relative importance of differential talent versus luck in creating inequality. The 1% difference in the mean return due to differential investment talent seems to be relatively small, but with time and the power of compounding it does create a large wealth differential. For example, with $n = 100$ investment periods and a \$1 initial investment, talented investors investing at 12% (with certainty) end up with wealth of \$83,522, while less talented investors investing at only 11% (with certainty) end up with only \$34,064. These

[11] We thank the anonymous referee for suggesting this analysis.

TABLE 2.—COMPARISON OF THE WEALTH DISTRIBUTION IN A HETEROGENEOUS TALENT POPULATION WITH THE PARETO AND THE LOG NORMAL DISTRIBUTIONS[a]

Standard Deviation of Talent Distribution, σ_μ (%)	Observed Statistic[b]	
	D_P	D_L
0.00	0.017	0.155
0.25	0.021	0.159
0.50	0.048	0.175
0.75	0.063	0.170
1.00	0.130	0.205
1.25	0.206	0.215
1.50	0.265	0.296

[a] For all investors returns are drawn randomly from a normal distribution with $\sigma = 20\%$. However, for each investor the mean of the return distribution is different. The means are taken as normally distributed around 12%. We report the Kolmogorov-Smirnov statistic for different values of the standard deviation of the distribution of the means. The first case, with a standard deviation of 0, is the homogeneous-talent case.

[b] D_P and D_L are the Kolmogorov-Smirnov statistics corresponding to the Pareto and log normal distributions respectively. With 1600 observations, the critical Kolmogorov-Smirnov value at the 1% confidence level is $D_{1\%} = 0.041$, at the 5% level it is $D_{5\%} = 0.034$, and at the 20% level it is $D_{20\%} = 0.027$. The highlighted cases are those in which the Pareto distribution cannot be rejected.

wealth values translate to a Gini wealth inequality coefficient of 0.21. The other possible source of inequality is the randomness of the wealth accumulation process (luck). In fact, the power of randomness in inducing inequality is typically much greater than the inequality induced by the 1% average return talent differential. In the simulation of equally talented investors drawing random returns from a normal distribution with mean 12% and $\sigma = 20\%$, the Gini coefficient after 100 periods is 0.71. When both differential talent and randomness are combined, in a simulation where $\mu_1 = 12\%$, $\mu_2 = 11\%$, and $\sigma_1 = \sigma_2 = 20\%$, the Gini coefficient after 100 investment periods is 0.75.[12] Thus, the effect of randomness is typically much stronger than the effect of modest talent differences, and it accounts for most of the wealth inequality.

It is common to compensate investment and fund managers for their performance. Compensation schemes for fund managers typically assume that the fund manager can improve the fund's performance by collecting information and by employing special investment skills. Under this assumption several works have analyzed the optimal compensation contract (see, for example, Roll, 1992; Stoughton, 1993; and Admati and Pfliederer, 1997). Our results suggest that there may be differential investment talents in the stock market, but to be consistent with the empirically observed Pareto distribution there must be only a relatively small difference in investment talent. This may seem to imply that a fund's performance is not linked with its management. However, recall that managers can affect performance even if they do not possess exceptional investment skills. For example, with their marketing skills managers can increase the amount of money under management even if they cannot achieve abnormal returns by stock picking or by market timing. Thus, one should distinguish between man-

[12] This figure is close to the empirical Gini coefficient in the United States, which is about 0.8 (Wolff, 2000).

agerial talent and investment ability. Differential managerial talent and differential efforts obviously do exist, and therefore managers should be compensated differentially. To be more specific, efficient management, cost reduction, good management of inventory, bargaining power, marketing and advertising talent, and of course, effort all affect performance. For such talent, management should be compensated, even if it is not capable of picking securities or timing the market better than anyone else.

The preceding theoretical and numerical analysis assume a simple wealth accumulation process [equation (2)]. In practice, the capital investment process is much more complex: it involves many important issues such as diversification across assets, market mechanisms, and complicated interactions between the various traders. To complement the theoretical analysis we conduct an experiment designed to investigate the development of the wealth distribution in a realistic market setting. In the next two sections we describe the experiment and the results. We find that some differential talent does exist, yet this differential is small. This allows the coexistence of a Pareto wealth distribution with slight differential talent, exactly as found in the numerical simulations.

III. The Experiment

The experiment was designed to create an atmosphere close to that of a real security market. The main contributors to this atmosphere were the ability to gain or lose out-of-pocket money, and investment over an extended period of time with many trading rounds.

There were 20 firms in the experiment, and subjects traded in the stocks of those firms. There were 10 trading rounds, one week apart, and in each round the market price of each stock was determined by supply and demand, exactly as in a marketplace with limit orders. After the trade, the book value of each firm was changed randomly, reflecting the firm's profit or loss from operation. In each round the book and market values might differ, exactly as occurs with closed-end mutual funds. However, after the tenth trading round, all the firms liquidated their assets and the market values were equal to the corresponding book values. Subjects traded at the beginning of the period, and the firm revealed its profit or loss from operation (book value) at the end of the period.

The subjects participating in the experiment could gain or lose real out-of-pocket money, depending on the realized returns on the investments which they selected.[13] The subjects were first-year MBA students who were taking a finance course at the Hebrew University of Jerusalem. The experiment was not mandatory, as loss of real out-of-pocket money was possible. Out of a potential 67 students, 63 chose to participate in the experiment. The subjects' average age was about 25, and most subjects were employed either part or full time. The information given below, as well as the trade procedure, was fully known to the subjects and to the experiment manager.[14] Any information which was not known in advance to the subjects (such as the future buy-sell orders for a given stock) was also unknown to the experiment manager.

Each subject was given an initial investment allotment of $30,000 in "experiment money," and could buy stocks of the 20 pure equity firms reported in table 3. The book value of each firm's assets at the beginning of the experiment is shown in the right-hand column of table 3. In each subsequent period, the book value of the firm's asset either grew or declined at random. The random variable which determined the book value was drawn from a normal distribution with the corresponding mean and variance reported in table 3 (which were known to the subjects). As there are ten periods (or trading rounds) in the experiment, the i^{th} firm's asset value at the end of the 10^{th} period is given by

$$V_{i,B}^{10} = V_{i,B}^{0} \prod_{t=1}^{10} (1 + \tilde{R}_{i,B}^{t}),$$

where $V_{i,B}^{t}$ is the book value of firm i in period t and $\tilde{R}_{i,B}^{t}$ is the random growth rate corresponding to firm i in period t.[15]

To facilitate the subject's portfolio choice, all the pairs of random variables $(\tilde{R}_{i,B}^{t}, \tilde{R}_{j,B}^{t})$ are pairwise independent (zero correlation), and each random variable $\tilde{R}_{i,B}^{t}$ is independent over time (this information was known to the subjects). To avoid differences between the firm's accounting profit and economic profit, the subjects were told that $\tilde{R}_{i,B}^{t}$ is the cash rate of return of the firm on the firm's assets, and not an accounting return. This makes the book value at the liquidation date equal to the market value (cash distributed to the subjects).

The subjects were told that at the end of the tenth round of trading all firms would liquidate and that the liquidation value would be $V_{i,B}^{10}$, that is, the book value at that time. The liquidation value of each firm would then be distributed to the subjects in accordance with the proportion of shares they held of each particular firm. Thus, this scenario is very similar to a trade in *term trusts*[16] (shares of a closed-end

[13] Creating a situation where losses are possible greatly affects the subjects' behavior. For example, when losses are not possible, subjects tend to take huge leverage (see Kroll, Levy, and Rapoport, 1988), but when losses are possible, subjects decrease their leverage (see Kroll and Levy, 1992).

[14] On the role of public information in explaining laboratory asset market data, see Smith (1962) and Copeland and Friedman (1991).

[15] Obviously, if the number of periods is very large ($n \to \infty$), Theorem 1 implies that if there is a lower bound on the book value, then $V_{i,B}^{n}$ converges to the Pareto distribution (or to the log normal distribution if there is no lower bound). However, note that the subjects' wealth is not $V_{i,B}^{n}$, and it is affected by the trades they conduct. They can accumulate wealth by buying the stock at a low market price and selling it at another trading round at a higher market price, irrespective of the book values. Thus, the subjects' wealth accumulation process is not the firms' book-value process. We thank the referee for clarifying this point.

[16] For more on term trusts, see, for example, "The Terminator," *Forbes*, August 11, 1997, p. 130.

TABLE 3.—THE BASIC DATA[a]

Stock Number	Industry	Name	Mean (%)	Standard Deviation (%)	Book Value at Time t_0 ($)
1	Food	Elite	3.0	4.0	78,930
2	Food	Shemen	4.0	6.0	70,160
3	Food	Tempo	5.0	8.0	59,198
4	Metal	Ordan	9.0	12.0	61,390
5	Metal	Htacof	4.0	7.0	51,546
6	Metal	Rav-Baraich	6.0	6.0	140,321
7	Industrial	Klal	12.0	14.0	64,433
8	Industrial	Andin	4.0	3.0	280,642
9	Industrial	Golan	5.0	10.0	37,886
10	Industrial	Mercazit	15.0	8.0	256,525
11	Industrial	Teva	7.0	9.0	77,956
12	Industrial	Yam-Amlach	9.0	12.0	61,390
13	Industrial	Electra	6.0	10.0	50,515
14	Oil	Joel	4.0	9.0	31,182
15	Oil	Hanal	8.0	9.0	93,547
16	Oil	Magen	10.0	12.0	70,160
17	Real estate	Marlaz	16.0	21.0	40,091
18	Real estate	Bet-Zave	8.0	8.0	118,396
19	Real estate	Yarden	15.0	10.0	164,176
20	Real estate	M.T.M.	10.0	14.0	51,546

[a] The risk-free interest rate $r = 2\%$.

mutual fund with a given predetermined terminal liquidation date), which can generally exhibit an observable premium or discount between their market value and the total value of their assets, but at the termination date the discount or premium vanishes.

The subjects could borrow or lend money at a risk-free interest rate of $r = 2\%$, and there were no constraints on borrowing or lending. The value of the interest rate, and the parameters of the firms' book growth, are chosen to roughly correspond to annual data. Thus, the period in the experiment roughly corresponds to one year.

The profit or loss of each subject at the termination date was determined as follows. Each subject received at time t_0 $30,000 in "experiment money" with which he/she can buy stocks or invest in the riskless asset. If the subject does not go bankrupt during the experiment, at the end of the tenth round the k^{th} investor's wealth is given by

$$X_k^{10} = \sum_{i=1}^{20} \frac{N_{i,k}}{N_i} V_{i,B}^{10} - B_k^9(1 + r),$$

where

X_k^{10} is the wealth of the k^{th} investor at the end of the tenth round,
$V_{i,B}^{10}$ is the liquidation value of the i^{th} firm at the end of the tenth period,
N_i is the number of shares issued by the i^{th} firm,
$N_{i,k}$ is the number of shares of the i^{th} firm held by the k^{th} investor, and
B_k^9 is the amount of money the k^{th} investor borrowed at the end of the ninth trading round. Thus, $B_k^9(1 + r)$ is the amount of money the subject should pay back to the bank at the end of the 10th trading round, where r denotes the interest rate. Note that if the subject lends money, then $B_k^9 < 0$; hence $-B_k^9(1 + r) > 0$ (and money is received from the bank).

To calculate the actual dollar reward of each subject, each $1,000 in experiment money represented $1 in actual money. At the end of *each* trading round, the net market value of the assets of each subject is examined. If it is negative, the subject goes bankrupt and pays money out of his/her pocket to the experiment manager.

The subjects were allowed to submit buy and sell limit orders for stocks, and the equilibrium price was determined by the intersection of the aggregate supply and demand curves.[17] There were no transaction costs on trading in the securities,[18] and short sales were not allowed. At the end of each trading round, each investor received information on his/her portfolio composition and his/her net wealth. Each firm's new book value, new stock price, and the number of shares traded were also provided after each round as public information. Subjects, however, did not have direct access to the composition of other subjects' portfolios.

[17] A call market, rather than a continuous auction market, is assumed. It is possible that market equilibrium would be achieved quicker with an auction market; however, in our experiment market equilibrium, and even strong support for the CAPM, are achieved rather quickly (see Levy, 1997).

[18] On considerations for including or excluding transaction costs from an experimental market see Plott and Smith (1978) and Forsythe, Palfrey, and Plott (1982).

IV. The Experimental Results

In section IV A we test the goodness of fit of the wealth distribution obtained in the experiment to the Pareto and to the log normal distributions. Finding a better fit to the Pareto distribution, we next estimate the value of the Pareto exponent α, and we measure the time it takes the distribution to converge to the Pareto distribution. The wealth distribution in the experiment can be affected by a different degree of risk aversion across subjects, by differential investment talent, and by luck. In section IV A we do not separate the various factors affecting the wealth distribution, but rather we analyze the shape of the wealth distribution obtained in the experiment, which may be due to a combined effect of all these factors.

In section IV B we test whether the main cause for inequality is differences in investment talent across investors, or simply luck.

A. *Goodness of Fit to the Pareto and the Log Normal Distributions*

The Pareto distribution can be written in various forms. Originally, it was formulated as follows (see Johnson & Kotz, 1970, chapter 19):

$$P(x) = \Pr(X \geq x) = \left(\frac{k}{x}\right)^{\alpha}, \quad \text{where} \quad k > 0, \quad \alpha > 0, \text{ and } x \geq k, \tag{8}$$

where $P(x)$ is the probability of the wealth being equal to or greater than x.[19] The cumulative distribution of wealth which follows from equation (8) is

$$F(x) = 1 - \left(\frac{k}{x}\right)^{\alpha}, \quad \text{where} \quad k > 0, \quad \alpha > 0, \quad x \geq k, \tag{9}$$

with a density function $f(x)$ given by

$$f(x) = \frac{\alpha k^{\alpha}}{x^{\alpha+1}}, \qquad \text{where} \quad \alpha > 0, \quad x > k > 0. \tag{10}$$

Pareto's law can also be formulated as

$$n = Ax^{-\alpha}, \tag{11}$$

where A is a constant, and n is the number of people having wealth x *or more*. Thus, n is the investor's rank by his/her wealth; the larger the wealth, the smaller the rank. Equation (11) can be also written as

$$x(n) = Cn^{-1/\alpha}, \tag{12}$$

where $x(n)$ is the wealth corresponding to the n-ranked individual, and $C = A^{1/\alpha}$. Taking logarithms of both sides of equation (12) yields

$$\log x(n) = \log C - \frac{1}{\alpha} \log n. \tag{13}$$

For these definitions and further analysis, see Johnson and Kotz (1970). The equivalence between equations (10) and (12) is shown in appendix A.

In this study we employ several methods to test whether the distribution of subjects' wealth as obtained in our experiment follows Pareto's law. Before we turn to the statistical significance test, let us first provide a general description of the experimental wealth distribution at the liquidation date. At the end of the experiment all the firms were liquidated and each subject received a reward according to his/her accumulated wealth. There were no bankruptcies in the experiment.[20] Therefore, each subject received a positive financial reward at the end of the experiment, and no one paid money to the experiment manager. There was a big dispersion of the reward. The highest reward of a subject was \$547 (corresponding to \$547,000 "experiment wealth") and the lowest was \$33 (corresponding to \$33,000 "experiment wealth").

Figure 1 shows the wealth distribution histogram at the liquidation date. Though the histogram intervals are determined arbitrarily (\$10,000 width), the long right tail is quite obvious; the wealth distribution is not symmetrical, and generally seems to be in good agreement with the Pareto distribution and possibly also with the log normal distribution.[21] However, these assertions have to be tested statistically. Therefore, we next test the relationship between the Pareto distribution and the wealth distribution as obtained in the experiment.

We first describe the use of equation (13) for this purpose. After each round of trade we measure the wealth of each subject, which is given by the market value of his/her portfolio. Then, we rank all the subjects by their wealth. Having (for each trading round) the pairs $(x(n), n)$, that is, the wealth of each subject and the corresponding rank n of that subject, we run the following regression, corresponding to equation (13):

$$\log x(n_i) = \hat{a} + \hat{b} \log n_i + e_i, \tag{14}$$

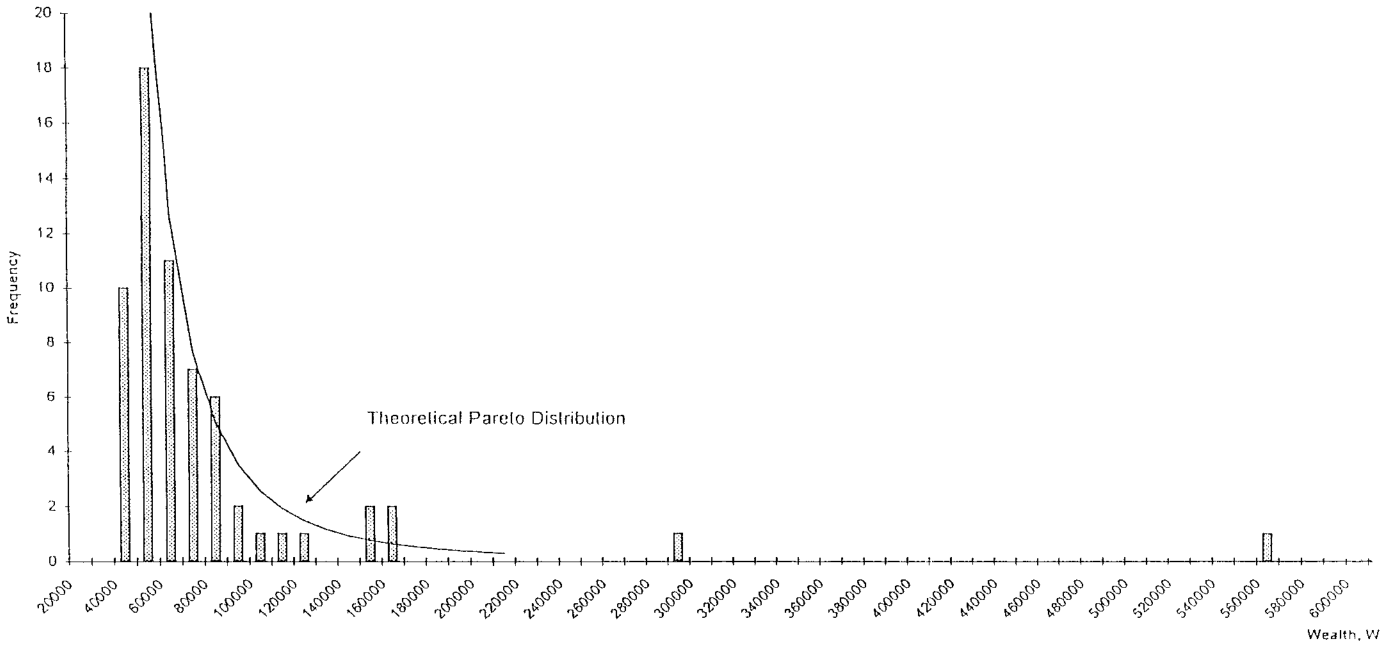
where $i = 1, 2, \ldots, 63$ (recall that there are 63 subjects). [For employing this method to test the goodness of fit of the

[19] As both wealth and normalized wealth follow a Pareto distribution, x can be taken as either of these. In the present context we take x as the absolute wealth.

[20] This may be the result of a self-imposed lower bound: becoming more conservative as one becomes poor.

[21] The empirical wealth distribution can typically be divided into two distinct regions: the low-medium wealth range, where the dominant factors affecting wealth are labor income and consumption, and the high wealth range, where the dominant factor is investment of capital. The Pareto distribution describes the empirical wealth distribution in the high wealth range. In this experiment, the wealth of all subjects (not just the wealthiest) was driven by capital investment; thus we expect the Pareto distribution to describe the wealth distribution at all wealth levels.

FIGURE 1.—TERMINAL WEALTH DISTRIBUTION



Pareto distribution, see also Ijiri and Simon (1977).[22]] If the subject's wealth is distributed according to Pareto's law, we expect to find a close linear fit [see equation (13)].

We employed the regression (14) to test whether the wealth distribution obeys Pareto's law. We ran eleven separate regressions, based on the wealth data as measured at the end of the ten trading rounds as well as the liquidation date. Table 4 reports the coefficients $\hat{b}$, the t-values of the coefficients, and the coefficients of correlation R^2 of these regression lines.

The fit to the wealth distribution of the Pareto distribution is striking, and in the last few trading rounds it is almost perfect. The slope is, of course, negative in all eleven rounds [as expected; see equation (14)]. The t-values range from -3.73 (in the first round) to about -47 in the last two rounds. After only three trading rounds, R^2 is greater than 90%. In rounds 4–11 the R^2 is very close to unity, indicating an almost perfect fit to the Pareto distribution after only four rounds. Notice that as the parameters of the returns in our experiment correspond to annual returns, this implies a convergence to the Pareto distribution after only a few years.

Figure 2 shows the regression line [corresponding to equation (14)] for the wealth distribution at the liquidation date, round 11. The regression lines for the first ten rounds are reported in appendix B. Note that the horizontal axis measures the subject's rank (1–63) by his/her wealth, and as there are 63 observations, the number ranges from 0 (log 1 for the wealthiest subject) to 1.799 (which is log 63, the rank of the poorest subject). The horizontal-axis range is therefore the same for all trading rounds. The vertical axis changes from one trading round to another because wealth changes across trading rounds.

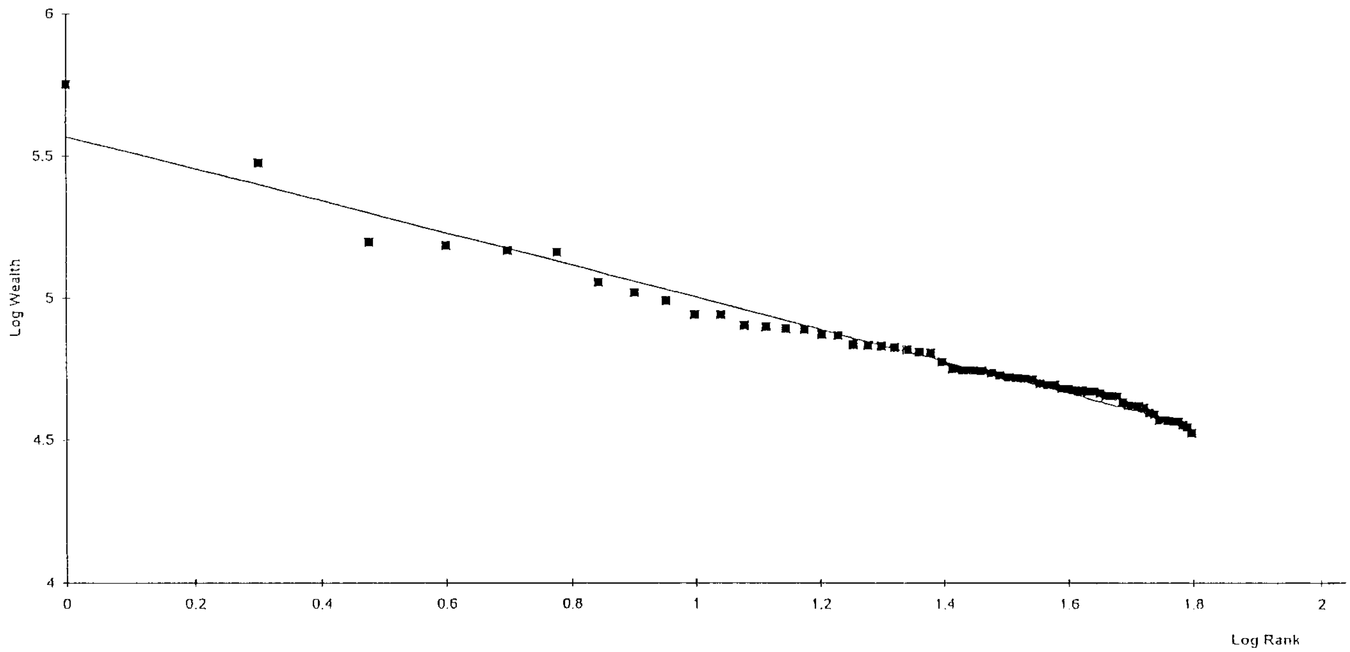
Contrasting equations (13) and (14), we see that $\hat{b}$ is an estimate of $-1/\alpha$. Therefore, to obtain the estimate of α we need to calculate $-1/\hat{b}$ for the values of $\hat{b}$ given in table 4. Note that $\hat{\alpha}$ tends to decrease as we advance in the trading rounds, and it is 1.85 and 1.78 for the tenth and eleventh rounds [that is, $-(1/-0.540)$ and $-(1/-0.563)$, respectively; see table 4]. It is interesting to note that $\hat{\alpha}$ for the empirical U.S. wealth distribution is estimated to be 1.35; for the United Kingdom, 1.06; and for France, 1.83 (see Wolff, 1996, and Fermi, 1998). Our $\hat{\alpha}$ estimate from the terminal wealth distribution is a little higher than the empirical values in the United States and the United Kingdom, and similar to the value in France. As $\hat{\alpha}$ shows a tendency to decline and to converge in our experiment, one may suspect that the experiment's $\hat{\alpha}$-values tend to converge to a

[22] For the pros and cons of various tests of the Pareto law see Aigner and Goldberger (1970), Harrison (1981), and Levy and Solomon (1997).

TABLE 4.—THE REGRESSION SLOPE AND THE R^2 OF WEALTH DISTRIBUTION [EQUATION (13) IN THE TEXT] AT THE ELEVEN TRADING ROUNDS

Trading Round	Coefficient $\hat{b}$ [Equation (13)]	$\hat{\alpha} = -1/\hat{b}$	t-value	R^2
1	-0.019	52.63	-3.73	0.186
2	-0.218	4.59	-15.76	0.803
3	-0.266	3.76	-26.34	0.919
4	-0.332	3.01	-31.41	0.942
5	-0.425	2.35	-35.65	0.954
6	-0.489	2.04	-43.80	0.969
7	-0.448	2.23	-41.36	0.965
8	-0.480	2.08	-42.68	0.967
9	-0.518	1.93	-44.96	0.971
10	-0.540	1.85	-47.88	0.974
11	-0.563	1.78	-47.39	0.973

FIGURE 2.—REGRESSION OF LOG TERMINAL WEALTH ON LOG RANK [EQUATION (13)]



value lower than 1.78, closer to those $\hat{\alpha}$'s observed in the United States and the United Kingdom.

The convergence of the experimental wealth distribution to the Pareto distribution is astonishing in its speed. What is the explanation for this rapid convergence? We believe that there are two main elements contributing to this result: the power of compounding and leverage. The subjects in the experiment took leveraged positions of up to 600% of their wealth. With such leverage the possible range of returns is very wide, and together with the power of compounding can create extreme wealth inequality very quickly. For example, suppose that all investors start with \$1 and that with leverage the highest possible rate of return is 50% and the lowest is -40%. After ten trading rounds the luckiest investor, with 10 high draws, will have wealth of 1.5^{10}, or \$57.7, and the unluckiest investor will have 0.6^{10}, or \$0.006. Thus, within only ten rounds the richest person will be about 10,000 times wealthier than the poorest person. To test this intuition about the importance of leverage and compounding we ran the following numerical experiment: we let investors invest in twenty stocks with realistic annual return distributions (the mean returns are in the range 3%–15%, and the standard deviations are in the range 3%–21%). At every period each investor randomly selects three of the twenty stocks and divides his investment equally among these three. All investors start with \$1. We investigate two alternative scenarios: in the first, investors cannot take leverage. In the second, we introduce a risk-free asset and allow investors to borrow up to 600% of their wealth at 2% interest (investors choose their investment proportion in the risk-free asset randomly from the range $[-6, 1]$; the maximal leverage actually taken in the experiment was 600%). The lower wealth bound is set at 30% of the average wealth. The resulting wealth distribution after 15 rounds is reported in figure 3. Whereas the no-leverage scenario (panel A) reveals a distribution which is still very different from the Pareto distribution (the straight line), the scenario with leverage reveals good agreement with the Pareto distribution after only 15 rounds. Thus, leverage and compounding indeed seem to be key elements contributing to the rapid convergence to the Pareto distribution.

It is possible that the experimental wealth distribution is in good agreement not only with the Pareto distribution but with other distributions as well. A natural candidate to compete with the Pareto distribution is the log normal distribution, which is also positively skewed. To compare the relative goodness of fit of two natural candidate distributions, we compare the theoretical and the empirical cumulative distributions and employ the Kolmogorov-Smirnov test. Thus, we contrast the goodness of fit of the Pareto and log normal distributions to the experimental distribution by comparing the experimental cumulative distribution with these two theoretical cumulative distributions. To draw the theoretical Pareto distribution, we need first to estimate $\hat{\alpha}$ and $\hat{k}$. However, by equation (9) we have

$$1 - F(x) = \left(\frac{k}{x}\right)^{\alpha}. \tag{15}$$

Hence,

$$\log[1 - F(x)] = \alpha \log k - \alpha \log x. \tag{16}$$

As $1 - F(x)$ and $\log x$ can be estimated from the experiment data (or empirically), one can run a simple regression (treating $\alpha \log k$ as constant) to obtain an estimate for α:

$$\hat{\alpha} = \frac{-N \sum_{i=1}^{N} \log x_i \log [1 - F(x_i)] + \left(\sum_{i=1}^{N} \log x_i\right)\left(\sum_{i=1}^{N} \log [1 - F(x_i)]\right)}{N \sum_{i=1}^{N} (\log x_i)^2 - \left(\sum_{i=1}^{N} \log x_i\right)^2} \tag{17}$$

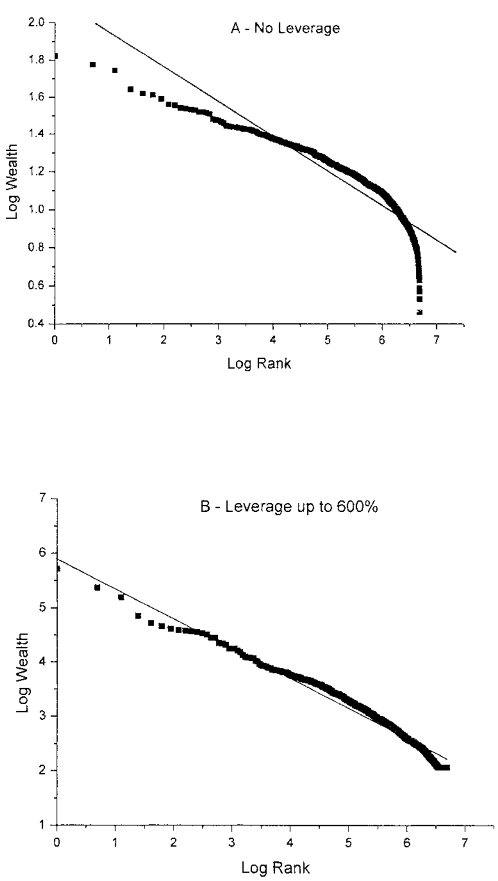
(see Johnson & Kotz, 1970, p. 235).

Having the estimate of $\hat{\alpha}$ and the arithmetic-mean values of the dependent and the independent variables, an estimate for the intercept $\alpha \log k$ is obtained [equation (16)]. As α is given by equation (17), and $\alpha \log k$ is given by equation (16), an estimate of $\hat{k}$ is also obtained. Using the wealth at each round, we can employ the above regression to estimate $\hat{\alpha}$ and $\hat{k}$ for each trading round.

To estimate the goodness of fit of the log normal distribution, we first estimate the mean and variance of log (wealth) given by $\hat{u} = \sum_{i=1}^{63} (\log x_i)/63$ and $\hat{\sigma}^2 = \sum_{i=1}^{63} (\log x_i - \hat{u})^2/63$.

Having $\hat{\alpha}$, $\hat{k}$, $\hat{u}$, and $\hat{\sigma}^2$, we can draw the theoretical cumulative distributions of the Pareto distribution and the log normal distribution corresponding to these parameters, as well as the experimental distribution. Figure 4 shows the theoretical Pareto distribution, the theoretical log normal distribution, and the experimental cumulative distribution corresponding to the terminal date. To test the hypothesis that the wealth distribution is not significantly different from the Pareto distribution or the log normal distribution we use the Kolmogrov-Smirnov statistic. For the Pareto distribution we find that the sample statistic is equal to $D = 0.1057$. With $n = 63$ observations, the 20% critical value is $D_c = 1.07/\sqrt{63} \cong 0.1348$ (and the 5% critical value is $1.36/\sqrt{63} \cong 0.1713$). Thus, the null hypothesis cannot be rejected even at a high significance level. The Pareto and the experimental distributions are so close to each other that even at a significance level which exceeds 20%, we cannot reject the null hypothesis asserting that the two distributions are equal, let alone with a more common significance level, such as 5%. This confirms our previous results revealing an excellent fit between the experimental wealth distribution and the Pareto distribution.

FIGURE 3.—THE WEALTH DISTRIBUTION WITH AND WITHOUT LEVERAGE



Employing the same statistical procedure, we also compare the experimental distribution with the log normal distribution. We find that the log normal distribution also fits the experimental wealth distribution rather well. However, the fit is not as good as with the Pareto distribution. With the log normal distribution, the null hypothesis asserting that the experimental distribution is a log normal distribution leads to $D = 0.1698$. With this value the log normal distribution is rejected at the 6% significance level,[23] let alone at the 20% level. Thus, the Pareto distribution describes the experimental distribution better than the log normal distribution, because the Pareto distribution hypothesis cannot be rejected even at 20% significance level, whereas the log normal distribution is rejected at the 6% significance level.

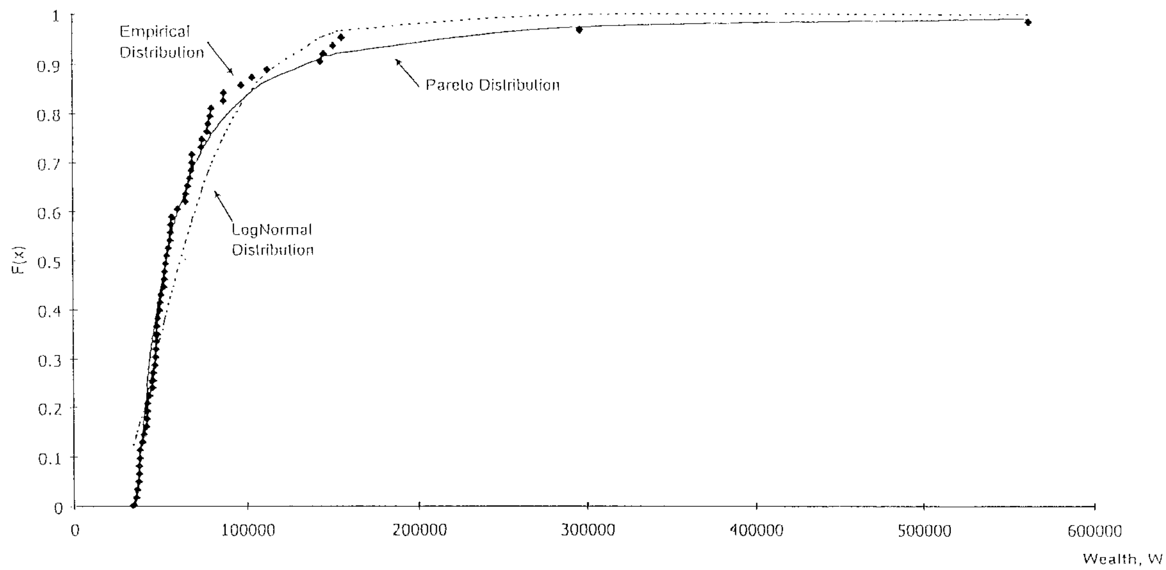
Finally, note that the estimates we obtain from equation (15) and (16) for $\hat{\alpha}$ and $\hat{k}$ for the wealth distribution in round 11 are $\hat{\alpha} \cong 1.728$ and $\hat{k} = 35{,}225$. Hence, with this estimation method [see equation (16)] we obtain a similar value of $\hat{\alpha}$ for the wealth distribution in the last round to the one we obtained with the rank-wealth estimation method [which yields $\hat{\alpha} = -(1/-0.563) = 1.776$; see table 4]. Again, the value obtained in the experiment is a bit larger than the corresponding $\hat{\alpha}$ for the United States and the United Kingdom, and a bit smaller than the one for France.

B. The Subjects' Investment Talent

An important implication of this study relates to the possible differences in investment talent across investors. The Pareto distribution implies rather extreme wealth inequality. How much of this wealth inequality is due to differential investment talent, and how much is due to luck?

[23] Calculated with the statistical programming language R (www.r-project.org).

FIGURE 4.—THE EMPIRICAL AND THEORETICAL CUMULATIVE DISTRIBUTIONS OF TERMINAL WEALTH



We have shown that substantial differential investment talent leads to a distribution of wealth which is different than the Pareto distribution (see the simulation results in section II). Thus, the fact that we find such a striking fit between the experimental wealth distribution and the Pareto distribution provides support, albeit not a proof, that luck is the main force driving the wealth differentiation generated by the investment process. Nevertheless, recall that, empirically as well as in the experiment, we do not obtain a *precise* Pareto distribution, and as we shall see, this finding is not inconsistent with the view that *some* differential investment talent may exist in the market.

The question whether there is significant differentiation in investment talent across investors is related to the issue of market efficiency. If the market is efficient, then no matter how talented the investor is in analyzing the available information, he/she will not be able to achieve abnormal returns. Though most empirical studies support the efficient-market hypothesis (EMH), there is no conclusive evidence regarding this issue. For evidence showing that the market is efficient, see Fama (1970, 1991). However, several market anomalies have been observed, which suggest that company specifics such as size or book-to-market ratio (Fama and French, 1992) or the stock's past performance (Jegadeesh and Titman, 1993) can be used to obtain abnormal returns. These anomalies imply that the market is inefficient and that talented investors may be able to outperform the market (adjusted for risk) by exploiting these anomalies. Moreover, even if the standard market efficiency tests reveal that the market *is* efficient, we still cannot safely conclude that there are no talented investors who can beat the market systematically, because it is possible that such investors use complex investment methods which are not tested for in the standard EMH tests.

How should investment talent be measured? Sharpe (1966) has devised a risk-adjusted measure, well-known as the Sharpe ratio or Sharpe index, in order to rank the performance of various portfolios or investment strategies in the mean-variance framework (see also Sharpe, 1994). Sharpe (1966) uses the performance measure he developed to test whether the performance of mutual funds in one period is indicative of the next period's performance (as we would expect if performance is driven by talent rather than luck). He measures the reward to volatility of mutual funds in two consecutive five-year periods. Sharpe finds a slight positive relationship between the performance in the two periods ($R^2 = 0.13$). This indicates that some differential investment talent of mutual funds managers exists. Recently, however, Beckers (1997) found that the major factor explaining mutual fund performance is luck. Samuelson (1989), who advocates market efficiency, asserts:

> Those lucky money managers who happen in any period to beat the comprehensive averages in total return seem primarily to have been merely lucky. Being on the honor roll in 1974 does not make you appreciably more likely to be on the 1975 honor roll. (Samuelson, 1989, p. 4).

Chevalier and Ellison (1999) find that there may be some differences in fund managers' performance; however, the differences in risk-adjusted annual returns are generally less than 1%. Thus, the issues of market efficiency and differential investment talent of fund managers are controversial.[24]

The studies cited above analyze the role of investment talent in the investment performance of mutual funds. A

[24] For more evidence regarding fund managers' abilities see Jensen (1968), Hendricks, Patel, and Zeckhauser (1993), Brown and Goetzmann (1995), Malkiel (1995), Gruber (1996), and Carhart (1997).

TABLE 5.—PERSISTENCE OF PERFORMANCE: REGRESSION COEFFICIENT t-VALUES OF EQUATION (18) FOR ALL ROUNDS $L \neq M$

L	$M = 1$	2	3	4	5	6	7	8	9	10	11
1		−13.26	−0.69	−2.93	−0.77	−0.16	1.23	−0.96	−1.22	−0.88	−1.33
2	−13.26		−1.02	3.63	0.25	−0.09	−0.63	−0.08	0.36	0.03	0.27
3	−0.69	−1.02		−1.99	0.43	−0.28	−0.19	−0.12	0.74	−0.29	0.56
4	−2.93	3.63	−1.99		6.29	0.57	−2.57	2.06	4.03	2.58	2.83
5	−0.77	0.25	0.43	6.29		0.75	−3.80	3.12	5.50	1.54	4.29
6	−0.16	−0.09	−0.28	0.57	0.75		−3.42	3.89	2.10	0.25	0.92
7	1.23	−0.63	−0.19	−2.57	−3.80	−3.42		−11.86	−6.76	−1.58	−6.17
8	−0.96	−0.08	−0.12	2.06	3.12	3.89	−11.86		7.55	0.69	5.48
9	−1.22	0.36	0.74	4.03	5.50	2.10	−6.76	7.55		2.36	4.23
10	−0.88	0.03	−0.29	2.58	1.54	0.25	−1.58	0.69	2.36		1.01
11	−1.33	0.27	0.56	2.83	4.29	0.92	−6.17	5.48	4.23	1.01	

parallel direct empirical analysis of the role of talent versus the role of luck in *personal* investment is generally very difficult, because the data regarding individuals' portfolios is usually not available. In the framework of our experiment, we have a unique opportunity to perform such a direct analysis.[25] The data we recorded include not only the wealth of each individual at the end of each trading round, but also the composition of his/her portfolio, and the rate of return achieved by each investor at each round.

The first method we employed in order to test for differential investment talent is simple: if differential talent exists, success in one trading round should (on average) predict success in other trading rounds. In this case, we would expect a positive autocorrelation of investment performance. However, if performance is driven by luck, as in flipping a coin, success in one period has no predictive power whatsoever regarding future performance. In each trading round we measure the rate of return obtained on the portfolio of the k^{th} investor, denoted by R_k. Then, we run the regression

$$R_{k,L} = a + bR_{k,M} + e_k, \tag{18}$$

where $k = 1, 2, \ldots, 63$, where $L \neq M$, and where $L = 1, 2, \ldots, 10$, $M = 1, 2, \ldots, 10$ stand for the ten trading rounds, and 11 stands for the final liquidation period. If an investor has exceptionally good investment talent with a relatively high rate of return in round M (relative, that is, to the return obtained by the other subjects), then a relatively high rate of return is expected also in round L. By this argument, if differential investment talent exists, we expect b to be positive. If b is negative, it implies that a success in round M predicts a failure in round L. If b is not significantly different than 0, we tend to conclude that success is due to pure luck.

Table 5 reports the regression coefficients for all pairs $L \neq M$ of the regression (18). The table reveals that there are more positive significant slopes b than negative ones, which tends to support the idea that some differential talent exists. Out of the 110 t-values reported in table 5, there are 30 significant positive values (at a 5% level), 17 significant negative values, and 63 nonsignificant values. Thus, throughout the experiment, it seems that the wealth was determined more by luck than by investment talent, because there are 80 out of 110 coefficients which are either not significantly different from 0 or are significantly negative. Nevertheless, the 30 significant positive values b indicate that some differential investment talent may exist. Table 6 complements table 5 by showing the relevant correlations R for all rounds $L \neq M$. As we can see from the table, the correlations are relatively low, with the lowest one ($R = -86\%$) corresponding to rounds 1 and 2, and the highest ($R = 69\%$) corresponding to rounds 8 and 9.

A possible explanation for the nonzero slopes b has to do with the different leverage policy adopted by the subjects rather than with differential stock selection ability. To illustrate, suppose that some investors tend to borrow more than other investors, and that this tendency does not change across time. Investors who are highly leveraged will generally have returns which are higher than the average when the market goes up, and lower than the average when the market goes down. If the stock market goes up, say in rounds 3 and 6, then we expect to have a positive slope corresponding to these two rounds due to the leverage effect, even though there is no differential stock selection ability at all. As the stock market (in reality, as well as in our experiment) generally goes up in more periods than it goes down, we should expect to find more positive (and significant) slopes b than negative slopes. This effect is also expected to be present in the actual financial markets; if there is no significant differentiation in investment talent, and the stock market goes up in more years than it goes down, we should expect to find more positive slopes than negative slopes because of the leverage effect.

In order to control for the differential leverage effect, as well as for the risk of the selected stocks, we refine the differential talent test by employing Sharpe's (1966) test. Sharpe's ratio R/V, which is a risk-adjusted index, measures the investment performance regardless of the adopted leverage; the higher the ratio, the better the risk-adjusted invest-

[25] Notice that the book values of each firm are determined randomly. Hence, talent is irrelevant if only book values are considered. However, talented investors can locate those securities with relatively low market values (as determined by all subjects in each round), yielding relatively high rates of return.

TABLE 6.—PERSISTENCE OF PERFORMANCE: PERFORMANCE CORRELATIONS FOR ALL ROUNDS $L \neq M$ [Equation (18)]

L	$M = 1$	2	3	4	5	6		8	9	10	11
1		−0.862	−0.087	−0.351	−0.098	−0.020	0.156	−0.122	−0.155	−0.113	−0.167
2	−0.862		−0.129	0.421	0.032	−0.012	−0.080	−0.010	0.046	0.004	0.034
3	−0.087	−0.129		−0.247	0.055	−0.036	−0.024	−0.016	0.094	−0.037	0.071
4	−0.351	0.421	−0.247		0.627	0.073	−0.312	0.255	0.458	0.314	0.341
5	−0.098	0.032	0.055	0.627		0.095	−0.438	0.371	0.576	0.193	0.481
6	−0.020	−0.012	−0.036	0.073	0.095		−0.402	0.446	0.260	0.032	0.117
7	0.156	−0.080	−0.024	−0.312	−0.438	−0.402		−0.835	−0.655	−0.198	−0.620
8	−0.122	−0.010	−0.016	0.255	0.371	0.446	−0.835		0.695	0.088	0.574
9	−0.155	0.046	0.094	0.458	0.576	0.260	−0.655	0.695		0.289	0.476
10	−0.113	0.004	−0.037	0.314	0.193	0.032	−0.198	0.088	0.289		0.128
11	−0.167	0.034	0.071	0.341	0.481	0.117	−0.620	0.574	0.476	0.128	

ment performance.[26] To test whether differential investment talent exists after the leverage and risk effects are neutralized, like Sharpe, we calculate the reward-to-volatility ratio twice: once for the first five trading rounds and once for the next six rounds. We run the regression

$$(R/V)_{k,2} = a + b(R/V)_{k,1} + e_k, \tag{19}$$

where $k = 1, 2, \ldots, 63$, subscripts 1 and 2 stand for the two subperiods, and R/V stands for Sharpe's performance ratio. Like Sharpe, we find a slight positive correlation with $R^2 = 0.05$, with nonsignificant slope ($t = 1.72$).

To sum up, there is no conclusive empirical evidence whether differential investment talent exists in the market. Sharpe's study of mutual funds and our experimental study reveal that although luck is a dominating factor determining performance, some differential talent may exist. The existence of differential talent is not consistent with a *precise* Pareto wealth distribution, but because empirically and experimentally one never finds a perfect fit to the Pareto distribution, some limited degree of differential talent may be consistent with the empirically observed approximate Pareto wealth distribution, as the simulations in section I demonstrate.

V. Concluding Remarks

Wealth distribution and wealth inequality are central issues in economics. Market efficiency and investment talent are important research topics in finance. In this paper we integrate these two research areas. We prove theoretically that if there is no differential investment talent (that is, the market is efficient in the sense that investors do not have stock selection or market timing abilities), the Pareto wealth distribution emerges. However, if substantial differential talent does exist, the resulting wealth distribution is not the Pareto distribution. These results, and the fact that the empirical wealth distribution in the high wealth range is in excellent agreement with the Pareto distribution (see, for example, Stiendl, 1965; Atkinson and Harrison, 1978; and Persky, 1992), lead us to suspect that substantial differential investment talent does not exist, and that chance is the main force responsible for the Pareto wealth distribution and the rather extreme inequality which it implies.

One may object to this argument on two grounds: First, as the theoretical results hold only in the limit of an infinite number of time periods, a chance-driven wealth accumulation process may not lead to the Pareto wealth distribution within a reasonable time frame (see Shorrocks, 1973). Second, although the empirical wealth distribution in the high wealth range is in excellent agreement with the Pareto distribution, it is not a mathematically perfect Pareto distribution. Thus, the observed distribution may be consistent with some degree of differential investment talent. To address these issues directly, we conducted an experimental study of wealth accumulation through investments in a capital market. Our experimental results are striking: we start with perfect equality (where each investor has \$30,000), and after only a few trading rounds (years) we obtain an extremely good fit to the Pareto distribution. The fit is supported both by the rank-wealth method, which yields a coefficient of correlation, R^2, of more than 97% from the ninth trading round (year) onwards, and by the Kolmogorov-Smirnov test, by which the Pareto distribution cannot be rejected even at a 20% significance level. Moreover, the parameter of the Pareto distribution, α, obtained in the experiment is similar in magnitude to the estimated parameter of the real wealth distribution in the United States, the United Kingdom, and France. Thus, even if one initially distributes wealth equally among investors, it takes only about one decade for the Pareto wealth distribution, and the strong inequality which it implies, to evolve. This leads one to suspect that the extreme inequality in modern western society is a very fundamental and robust outcome of the nature of the capital investment process. This inequality can emerge within only a few years—it is not necessarily the cumulative result of many generations of wealth accumulation.

Are there indeed differences in investment talent? Sharpe (1966) claims that mutual fund managers do reveal some differential investment talent. Other researchers report opposite results. Our experimental framework gives us a unique opportunity to go further and test the hypothesis of

[26] Sharpe's reward-to-volatility ratio is given by $R/V = (\mu - r)/\sigma$, where μ is the portfolio's average return, σ is the standard deviation, and r is the risk-free interest rate.

differential investment talent directly. If talent plays a significant role in the investment process, one would expect good performance in one period to be indicative (on average) of high investment talent, and therefore to be followed by more good performance in another period (again, on average). In our experiment we find that there may be slight differential investment talent across investors. Thus, the results generally support the efficient-market hypothesis, albeit not perfectly, as we cannot completely rule out any differential investment talent. Can slight differential investment talent (and slight market inefficiency) be consistent with the empirically observed Pareto wealth distribution? We find that one may have up to 1% difference in average (risk-adjusted) annual returns due to talent difference without rejecting the Pareto distribution. Thus, the proximity of the empirically and experimentally observed wealth distributions to the Pareto distribution is consistent with a small degree of differential investment talent, but puts a limit on this differential. Within this limit, of about 1% difference in risk-adjusted returns, we show that the main force driving inequality is the randomness of the investment process, rather than differential talent.

REFERENCES

Admati, Anat, and Paul Pfliderer, "Does It All Add Up? Benchmarks and the Compensation of Active Portfolio Managers," *Journal of Business* 70:3 (1997), 323–350.

Aigner, Dennis J., and Arthur S. Goldberger, "Estimation of Pareto's Law from Grouped Observations," *Journal of the American Statistical Association* 65:330 (1970), 712–723.

Aiyagari, S. Rao, "Uninsured Idiosyncratic Risk and Aggregate Saving," *Quarterly Journal of Economics* 109 (1994), 659–684.

Arnold, Barry, *Pareto Distributions* (Burtonsville, MD: International Cooperative Publishing House, 1983).

Atkinson, Anthony B., and Alan J. Harrison, *Distribution of Total Wealth in Britain* (Cambridge, U.K.: Cambridge University Press, 1978).

Auerbach, Alan J., and Laurence J. Kotlikoff, *Dynamic Fiscal Policy* (New York: Cambridge University Press, 1987).

Beckers, Stan, "Manager Skill and Investment Performance: How Strong is the Link?" *The Journal of Portfolio Management* 23:4 (1997), 9–23.

Blinder, Alan S., *Towards an Economic Theory of Income Distribution* (Cambridge, MA: MIT Press, 1974).

Boissevain, C. Harry, "Distribution of Abilities Depending upon Two or More Independent Factors," *Metron* 13 (1939), 49–58.

Boltzmann, Ludwig, *Lectures on Gas Theory* (Berkeley: University of California Press, 1964).

Brown, Stephen J., and William N. Goetzmann, "Performance Persistence," *Journal of Finance* 50 (1995), 679–698.

Carhart, Mark M., "On Persistence in Mutual Fund Performance," *Journal of Finance* 52 (1997), 57–82.

Castaneda, Ana, Javier Díaz-Giménez, and José-Victor Ríos-Rull, "Unemployment Spells, Cyclically Moving Factor Shares and Income Distribution Dynamics," Federal Reserve Bank of Minneapolis manuscript (1997).

Champernowne, David G., "A Model of Income Distribution," *Economic Journal* 63 (1953), 318–351.

Chevalier, Judith, and Glenn Ellison, "Are Some Mutual Fund Managers Better Than Others? Cross-Sectional Patterns in Behavior and Performance," *Journal of Finance* 54 (1999), 875–899.

Choquet, Gustave, "Le théorème de representation intégrale dans les ensembles convexes compacts," *Annales de l'Institut Fourier (Grenoble)* 10 (1960), 333–344.

Copeland, Thomas, and Daniel Friedman, "Partial Revelation of Introduction in Experimental Asset Markets," *Journal of Finance* 46 (1991), 265–295.

Davis, Harold, *The Analysis of Economic Time Series* (San Antonio, TX: The Principia Press of Trinity University, 1963). Originally Cowles Commission for Research in Economics monograph no. 6 (1941).

Fama, Eugene F., "Efficient Capital Markets: A Review of Theory and Empirical Work," *Journal of Finance* 25 (1970), 383–417.

——— "Efficient Capital Markets: II," *Journal of Finance* 46 (1991), 1575–1617.

Fama, Eugene F., and Kenneth French, "The Cross-Section of Expected Stock Returns," *Journal of Finance* 47 (1992), 427–465.

Feldstein, Martin, and Anthony Pellechio, "Social Security and Household Wealth Accumulation: New Microeconometric Evidence," *Review of Economics and Statistics* 90 (1979), 361–368.

Fermi, D. (Ed.), *QUID* (Paris: RTL, 1998).

Feynman, Richard P., Robert B. Leighton, and Matthew Sands, *The Feynman Lectures on Physics*, vol. 1 (Reading, MA: Addison Wesley, 1964).

Forsythe, Robert, Thomas R. Palfrey, and Charles R. Plott, "Asset Valuation in an Experimental Market," *Econometrica* 50 (1982), 537–567.

Furstenberg, Harry, "Translation-Invariant Cones of Functions on Semisimple Lie Groups," *Bulletin of the American Mathematical Society* 71 (1965), 271–326.

Gabaix, Xavier, "Zipf's Law for Cities: An Explanation," *Quarterly Journal of Economics* 114:3 (1999), 739–767.

Gruber, Martin J., "Another Puzzle: The Growth in Actively Managed Mutual Funds," *Journal of Finance* 51 (1996), 783–810.

Harrison, Alan, "Earnings by Size: A Tale of Two Distributions," *Review of Economic Studies* 48 (1981), 621–631.

Hendricks, Darryll, Jayendu Patel, and Richard Zeckhauser, "Hot Hands in Mutual Funds: Short-Run Persistence of Performance 1974–1988," *Journal of Finance* 48 (1993), 93–130.

Huggett, Mark, "Wealth Distribution in Life-Cycle Economies," *Journal of Monetary Economics* 38 (1996), 953–969.

Ijiri, Yuji, and Herbert Simon, *Skew Distributions and the Sizes of Business Firms* (North-Holland Publishing Company, 1977).

Jensen, Michael C., "The Performance of Mutual Funds in the Period 1945–1964," *Journal of Finance* 23 (1968), 389–416.

Jegadeesh, Narasimhan, and Sheridan Titman, "Returns to Buying Winners and Selling Losers: Implications for Stock Market Efficiency," *Journal of Finance* 48 (1993), 65–91.

Johnson, Norman L., and Samuel Kotz, *Continuous Univariate Distributions* (New York: John Wiley & Sons, 1970).

Kesten, Harry, "Random Difference Equations and Renewal Theory for Products of Random Matrices," *Acta Mathematica* 81 (1973), 207–248.

Kroll, Yoram, and Haim Levy, "Further Tests of the Separation Theorem and the Capital Asset Pricing Model," *American Economic Review* 82:3 (1992), 664–670.

Kroll, Yoram, Haim Levy, and Amnon Rapoport, "Experimental Tests of the Separation Theorem and the Capital Asset Pricing Model," *American Economic Review* 78 (1988), 500–519.

Levy, Haim, "Absolute and Relative Risk Aversion: An Experimental Study," *Journal of Risk and Uncertainty* 8 (1994), 289–307.

——— "Risk and Return: An Experimental Analysis," *International Economic Review* 38:1 (1997), 119–149.

Levy, Moshe, "Are Rich People Smarter?," *Journal of Economic Theory* (forthcoming).

Levy, Moshe, and Sorin Solomon, "Power Laws are Logarithmic Boltzmann Laws," *International Journal of Modern Physics C* 7 (1996), 595–601.

——— "New Evidence for the Power-Law Distribution of Wealth," *Physica A* 242 (1997), 90–94.

Lintner, John, "Security Prices, Risk, and Maximal Gains from Diversification," *Journal of Finance* 20 (1965), 587–616.

Loomis, L., "Unique Direct Integral Decompositions on Convex Sets," *American Journal of Mathematics* 84 (1962), 509–526.

Malkiel, Burton G., "Returns from Investing in Equity Mutual Funds 1971–1991," *Journal of Finance* 50 (1995), 549–572.

Mandelbrot, Benoit, "The Pareto-Levy Law and the Distribution of Income," *International Economic Review* 1 (1960), 79–106.

Pareto, Vilfredo, *Cours d'Economique Politique*, vol. 2 (1897).

——— *Manual of Political Economy* (New York: Augustus M. Kelly, 1971). Translated from *Manuale d'Economia Politica* (1906).

Persky, Joseph, "Retrospectives: Pareto's Law," *Journal of Economic Perspectives* 6:2 (1992), 181–192.

Plott, Charles R., and Vernon L. Smith, "An Experimental Examination of Two Exchange Institutions," this REVIEW 45 (1978), 133–153.
Ríos-Rull, José-Victor, "Models with Heterogeneous Agents" (pp. 98–125), in T. F. Cooley (Ed.), *Frontiers of Business Cycle Research* (Princeton, NJ: Princeton University Press, 1995).
Roll, Richard, "A Mean/Variance Analysis of Tracking Error," *Journal of Portfolio Management* 8:4 (1992), 13–22.
Samuelson, Paul A., "The Judgment of Economic Science on Rational Portfolio Management: Indexing, Timing, and Long-Horizon Effects," *Journal of Portfolio Management* 16 (Fall 1989), 4–12.
Sharpe, William F., "Mutual Fund Performance," *Journal of Business* 39 (1966), 119–138.
——— "The Sharpe Ratio," *Journal of Portfolio Management* 21:1 (1994), 49–58.
Shorrocks, Anthony F., *Aspects of the Distribution of Personal Wealth*, London School of Economics PhD dissertation (1973).
Simon, Herbert, "On a Class of Skew Distributions," *Biometrica* 42 (1955), 425–440. Reprinted in H. Simon, *Models of Man* (New York: John Wiley and Sons, 1957).
Slottje, Daniel J., *The Structure of Earnings and the Measurement of Inequality in the U.S.* (New York: Elsevier Science Publishers, 1989).
Smith, Vernon L., "An Experimental Study of Comparative Market Behavior," *Journal of Political Economy* 70 (1962), 111–137.
Stiendl, Josef, *Random Processes and the Growth of Firms—A Study of the Pareto Law* (London: Charles Griffin & Company, 1965).
Stoughton, Neal, "Moral Hazard and the Portfolio Management Problem," *Journal of Finance* 48:5 (1993), 2009–2028.
Thaler, Richard, "Mental Accounting and Consumer Choice," *Marketing Science* 4:3 (1985), 199–214.
Wold, Herman, and Peter Whittle, "A Model Explaining the Pareto Distribution of Wealth," *Econometrica* 25 (1957), 591–595.
Wolff, Edward, *Trends in Household Wealth during 1989–1992*, submitted to the Department of Labor (New York University, 1996).
——— "What Happened to Stock Ownership in the United States?" New York University working paper (2000).
Zipf, George K., *Human Behavior and the Principle of Least Effort* (Cambridge, MA: Addison-Wesley, 1949).

APPENDIX A

Derivation of the Rank-Wealth Relationship [Equation (12)]

The relationship

$$x(n) = Cn^{1/\alpha} \tag{A1}$$

holds if and only if the wealth probability density function is given by the Pareto law

$$f(x) = \frac{\alpha k^{\alpha}}{x^{\alpha+1}} \qquad (\alpha > 0, \quad x > k > 0), \tag{A2}$$

where n is the rank, x is wealth, the constant C is given by $C = kN^{1/\alpha}$, and N is the total number of individuals in the population.

(A-2) $\Rightarrow$ (A-1): Assuming the Pareto law (A-2), the number of individuals with wealth exceeding x is given by

$$n(x) = \left(\frac{x}{C}\right)^{-\alpha}.$$

The wealth of the n^{th} wealthiest individual is therefore given by

$$x(n) = kN^{1/\alpha} n^{-1/\alpha}.$$

(A-1) $\Rightarrow$ (A-2): The relation (A-1) can be written as

$$n(x) = \left(\frac{x}{C}\right)^{-\alpha}.$$

The number of individuals with wealth in the range $(x, x + dx)$ is $-dn/dx$ (as wealth increases, the rank decreases). Thus, the probability density function $f(x)$ is given by

$$f(x) = -\frac{1}{N}\frac{dn}{dx}.$$

Writing the derivative dn/dx explicitly [from equation (A-1)], and using the definition of C, $(C = kN^{1/\alpha})$, we obtain

$$f(x) = \frac{1}{N}\frac{\alpha}{C^{-\alpha}}\, x^{-(1+\alpha)} = \frac{1}{N}\frac{\alpha}{(kN^{1/\alpha})^{-\alpha}}\, x^{-(1+\alpha)} = \frac{\alpha k^{\alpha}}{x^{\alpha+1}}.$$

APPENDIX B

The Regression of Log Wealth on Log Rank for Trading Rounds 1–10

FIGURE B1.